Nonlocal corrections to the Boltzmann equation for dense Fermi systems

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A kinetic equation which combines the quasiparticle drift of Landau's equation with a dissipation governed by a nonlocal and noninstant scattering integral in the spirit of Snider's equation for gases is derived. Consequent balance equations for the density, momentum and energy include quasiparticle contributions and the second order quantum virial corrections. The medium effects on binary collisions are shown to mediate the latent heat, i.e., an energy conversion between correlation and thermal energy. An implementation to heavy ion collisions is discussed.

I. INTRODUCTION

One of the goals of the recent nuclear physics is to find the equation of state of nuclear matter. Indeed, the dependence of the pressure on the density of nucleons is a crucial input for a hydrodynamical modeling of heavy ion collisions or of astrophysical events like the big bang, supernova explosions and neutron stars [1].

In the absence of any direct measurement, it is hoped that the equation of state can be deduced from heavy ion collisions via the following scheme. Heavy ion collision data are fitted with the Boltzmann equation (BE)

$$
\begin{aligned}
&\frac{\partial f_1}{\partial t} + \frac{\partial \varepsilon_1}{\partial k}\frac{\partial f_1}{\partial r} - \frac{\partial \varepsilon_1}{\partial r}\frac{\partial f_1}{\partial k} \\
&= \sum_b \int \frac{dp\,dq}{(2\pi)^5} \delta\left(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4\right) \\
&\times |T|^2\left(\varepsilon_1 + \varepsilon_2, k, p, q, t, r\right) \\
&\times \Big[f_3 f_4 \left(1 - f_1\right)\left(1 - f_2\right) - \left(1 - f_3\right)\left(1 - f_4\right) f_1 f_2\Big]. \quad (1)
\end{aligned}
$$

Arguments of distributions f and energies ε are shortened as $f_1 \equiv f_a(k, r, t)$, $f_2 \equiv f_b(p, r, t)$, $f_3 \equiv f_a(k-q, r, t)$, and $f_4 \equiv f_b(p+q, r, t)$, with momenta k, p, q, coordinate r, time t, and spin and isospin a, b. Once the differential cross sections $|T|^2$ and the functional dependence of energy ε on the distribution f are fitted, the equation of state is evaluated from the kinetic equation.

This scheme has two drawbacks. First, accessible fits of the quasiparticle energy ε are not sufficiently reliable since two possible fits, momentum-dependent and momentum-independent, result in very contradictory predictions giving hard and soft equations of state, respectively [2]. When this more or less technical problem is resolved in future, one has to face the second drawback: the BE is not thermodynamically consistent with virial corrections to the equation of state. This problem is principal for "how can one infer the equation of state from the BE if the two equations are not consistent?". A consistency between the kinetic and the thermodynamic theories is a general question for the quantum statistics

exceeding the merits of the nuclear matter. Here we approach this question from nonequilibrium Green's functions. It is shown that the consistency is achieved by a consistent treatment of the quasiclassical limit which results in nonlocal and noninstant corrections to the scattering integral of the BE.

The need of nonlocal corrections can be seen on the classical gas of hard spheres. In the scattering integral of (1), all space arguments of the distributions are identical, i.e., colliding particles a and b are at the same space point r. In reality, these particles are displaced by the sum of their radii. This inconsistency has been noticed by Enskog [3] and cured by nonlocal corrections to the scattering integral. The equation of state evaluated from the kinetic equation with the nonlocal scattering integral is of the van der Waals type covering the excluded volume [3,4]. For nuclear matter, Enskog's corrections has been first discussed by Malfliet [5] and recently implemented by Kortemayer, Daffin and Bauer [6].

The noninstant corrections are closer to the chemical picture of reacting gases. In the scattering integral of (1), all time arguments of the distributions are identical what implies that the collision is instant. In reality, the collision has a finite duration which might be quite long when two particles form a resonant state. The resonant two-particle state behaves as an effective short-living molecule. Like in reacting gases [4], the presence of these molecules reduces the pressure since it reduces the number of freely flying particles. The finite duration of nucleon-nucleon collisions and its thermodynamic consequences has been for the first time discussed only recently by Danielewicz and Pratt [7]. The noninstant scattering integral and its consequencies for the linear response has been also discussed for electrons in semiconductors scattered by resonant levels [8].

Except for dense Fermi systems, the above intuitively formulated nonlocal and noninstant corrections has been confirmed by systematic approaches. For classical gases, this theory was developed already by Bogoliubov and Green [9,10]. Obtained gradient contributions to the scattering integral are the lowest order terms of the virial

expansion in the kinetic equation [11]. The first quantum kinetic equation with nonlocal corrections has been derived by Snider [12]. Recently, it has been recognized that Snider's equation is not consistent with the second order virial corrections to equations of state. A consistent quantum mechanical theory of the virial corrections to the BE has been developed from the multiple scattering expansion in terms of Møller operators [13] and confirmed by Balescu's formalism [14].

Presented treatment extends the nonlocal and non-instant corrections to dense Fermi systems. We follow Baerwinkel [15] in starting from nonequilibrium Green's functions and keeping all gradient contributions to the scattering integral. Baerwinkel's results are limited to low densities (to avoid medium effects on binary collisions) and not consistent (since he uses the quasiparticle approximation). Here we describe the binary collisions by the Bethe-Goldstone T-matrix which includes the medium effects. Instead of the quasiparticle approximation, the *extended* quasiparticle approximation is used. This extension is sufficient to gain consistency of the kinetic theory with the virial corrections to thermodynamic quantities.

II. EXTENDED QUASIPARTICLE PICTURE

We start our derivation of the kinetic equation from the quasiparticle transport equation first obtained by Kadanoff and Baym [16,17]

$$
\frac{\partial f_1}{\partial t} + \frac{\partial \varepsilon_1}{\partial k}\frac{\partial f_1}{\partial r} - \frac{\partial \varepsilon_1}{\partial r}\frac{\partial f_1}{\partial k} = z_1(1-f_1)\Sigma^<_{1,\varepsilon_1} - z_1 f_1 \Sigma^>_{1,\varepsilon_1}.
$$
(2)

Like in (1), quasiparticle distribution f, quasiparticle energy ε and wave-function renormalization z are functions of time t, coordinate r, momentum k and spin and isospin a. Self-energy Σ, taken from nonequilibrium Green's function in the notation of Kadanoff and Baym [16], is moreover a function of energy ω, however, it enters the transport equation only by its value at pole $\omega = \varepsilon_1$.

Particular forms of the quasiparticle energy and the scattering integral we derive for a model and an approximation used in nuclear matter for heavy ion collisions in the non-relativistic energy domain. The system is composed of protons and neutrons of equal mass m. They interact via an instant potential V. We assume no spin-flipping mechanism. As common, the self-energy is constructed from the two-particle T-matrix T^R in the Bethe-Goldstone approximation [16,18] as $[T^R_{sc}(1,2,3,4) = (1-\delta_{a_1a_2})T^R(1,2,3,4) + \frac{1}{\sqrt{2}}\delta_{a_1a_2}(T^R(1,2,3,4) - T^R(1,2,4,3))]$

$$
\Sigma^<(1,2) = T^R_{sc}(1,\bar{3};\bar{5},\bar{6})T^A_{sc}(\bar{7},\bar{8};2,\bar{4})
$$
$$
\times G^>(\bar{4},\bar{3})G^<(\bar{5},\bar{7})G^<(\bar{6},\bar{8}),
$$
(3)

and $\Sigma^>$ is obtained from (3) by an interchange $> \leftrightarrow <$. Here, G's are single-particle Green's functions, numbers are cumulative variables, $1 \equiv (t_1, r_1, a_1)$, and bars denote internal variables that are integrated over. Before (3) is plugged in (2), it has to be transformed into the mixed representation, [off-diagonal elements in spin and isospin are excluded, $a_1 = a_2 = a$]

$$
\Sigma^<(1,2) = \int \frac{d\omega}{2\pi}\frac{dk}{(2\pi)^3} e^{ik(r_1-r_2)-i\omega(t_1-t_2)}
$$
$$
\times \Sigma^<_a(\omega, k, r, t)_{r=\frac{r_1+r_2}{2}, t=\frac{t_1+t_2}{2}},
$$
(4)

and all Green's functions in (3), too.

The self-energy Σ is a functional of Green's functions G. This functional $\Sigma[G]$ is converted to the functional of the quasiparticle distribution $\Sigma_\varepsilon[f]$ via the extended quasiparticle approximation [17,19] $[z_1 = 1 + \frac{\partial}{\partial\omega}\mathrm{Re}\Sigma_{1\omega}|_{\varepsilon_1}]$

$$
G^{\gtrless}_{1,\omega} = \begin{pmatrix} 1-f_1 \\ f_1 \end{pmatrix} 2\pi z_1\delta(\omega-\varepsilon_1) + \mathrm{Re}\frac{\Sigma^{\gtrless}_{1,\omega}}{(\omega-\varepsilon_1)^2},
$$
(5)

where $G_{1,\omega} \equiv G_a(\omega, k, r, t)$ and similarly Σ. Unlike the plain quasiparticle approximation (without the second term) used by Baerwinkel [15], approximation (5) leads to the consistent theory. The first term brings the on-shell quasiparticle part, the second term is the off-shell contribution.

The off-shell part plays four-fold role. First, it justifies the kinetic equation (2). Equation (2) has been originally derived from the plain quasiparticle approximation neglecting the off-shell drift.* The off-shell part of $G^<$ in (5) compensates the off-shell drift so that (2) is recovered without uncontrollable neglects [17]. Second, in the quasiparticle energy $\varepsilon_1 = \frac{k^2}{2m_a} + \mathrm{Re}\Sigma^R_{1,\varepsilon_1}$, the off-shell part brings contributions that are essential for the correct binding energy [20]. Third, (5) provides Wigner's distribution $\rho = \int \frac{d\omega}{2\pi}G^<$ as a functional of the quasiparticle distribution f [17,20]. Fourth, in the scattering integral of (2), the off-shell part results in sequential three-particle processes with the off-shell propagation between the two composing binary processes. Since the three-particle processes are beyond the scope of the present paper, they are excluded from scattering integral.

III. NON-LOCAL SCATTERING INTEGRAL

Now the approximation is specified and we can start to simplify the scattering integral. In contrast to previous treatments of degenerated systems, we keep all terms

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*The well known term $[\mathrm{Re}G, \Sigma^<]$.

linear in gradients. The gradient expansion of the self-energy (3) is a straightforward but tedious task. It results in a one nongradient and nineteen gradient terms that are analogous to those found within the chemical physics [13,14]. All these terms can be recollected into a nonlocal and noninstant scattering integral that has an intuitively appealing structure of the scattering integral in the BE (1) with Enskog-type shifts of arguments.[†] In agreement with [13,14], all gradient corrections result proportional to derivatives of the scattering phase shift $\phi = \mathrm{Im}\,\ln T_{\mathrm{sc}}^R(\Omega, k, p, q, t, r)$,

$$
\begin{aligned}
\Delta_t &= \left.\frac{\partial \phi}{\partial \Omega}\right|_{\varepsilon_1 + \varepsilon_2} & \Delta_2 &= \left(\frac{\partial \phi}{\partial p} - \frac{\partial \phi}{\partial q} - \frac{\partial \phi}{\partial k}\right)_{\varepsilon_1 + \varepsilon_2} \\
\Delta_E &= \left.-\frac{1}{2}\frac{\partial \phi}{\partial t}\right|_{\varepsilon_1 + \varepsilon_2} & \Delta_3 &= \left.-\frac{\partial \phi}{\partial k}\right|_{\varepsilon_1 + \varepsilon_2} \\
\Delta_K &= \left.\frac{1}{2}\frac{\partial \phi}{\partial r}\right|_{\varepsilon_1 + \varepsilon_2} & \Delta_4 &= -\left(\frac{\partial \phi}{\partial k} + \frac{\partial \phi}{\partial q}\right)_{\varepsilon_1 + \varepsilon_2}.
\end{aligned}
\tag{6}
$$

After derivatives, Δ's are evaluated at the energy shell $\Omega \to \varepsilon_1 + \varepsilon_2$. The corrected BE with the collected gradient terms then reads $[\Delta_r = \frac{1}{4}(\Delta_2 + \Delta_3 + \Delta_4)]$

$$
\begin{aligned}
&\frac{\partial f_1}{\partial t} + \frac{\partial \varepsilon_1}{\partial k}\frac{\partial f_1}{\partial r} - \frac{\partial \varepsilon_1}{\partial r}\frac{\partial f_1}{\partial k} \\
&= \sum_b \int \frac{dp\,dq}{(2\pi)^5} \delta\left(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 + 2\Delta_E\right) \\
&\times z_1 z_2 z_3 z_4 \left(1 - \frac{1}{2}\frac{\partial \Delta_2}{\partial r} - \frac{\partial \bar{\varepsilon}_2}{\partial r}\frac{\partial \Delta_2}{\partial \omega}\right) \\
&\times |T_{\mathrm{sc}}^R|^2 \left(\varepsilon_1 + \varepsilon_2 - \Delta_E, k - \frac{\Delta_K}{2}, p - \frac{\Delta_K}{2}, q, r - \Delta_r, t - \frac{\Delta_t}{2}\right) \\
&\times \left[f_3 f_4 \left(1 - f_1\right)\left(1 - f_2\right) - \left(1 - f_3\right)\left(1 - f_4\right) f_1 f_2\right].
\end{aligned}
\tag{7}
$$

Unlike in (1), the subscripts denote shifted arguments: $f_1 \equiv f_a(k, r, t)$, $f_2 \equiv f_b(p, r - \Delta_2, t)$, $f_3 \equiv f_a(k - q - \Delta_K, r - \Delta_3, t - \Delta_t)$, and $f_4 \equiv f_b(p + q - \Delta_K, r - \Delta_4, t - \Delta_t)$.

The Δ's are effective shifts and they represent mean values of various nonlocalities of the scattering integral. These shifts enter the scattering integral in the form

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[†]The basic idea of the recollection can be demonstrated on the following rearrangement of the gradient approximation of a matrix product $C(1, 2) = A(1, \bar{3})B(\bar{3}, 2)$. In the mixed representation $C = AB + \frac{i}{2}\left(\frac{\partial A}{\partial \omega}\frac{\partial B}{\partial t} - \frac{\partial A}{\partial t}\frac{\partial B}{\partial \omega} - \frac{\partial A}{\partial k}\frac{\partial B}{\partial r} + \frac{\partial A}{\partial r}\frac{\partial B}{\partial k}\right)$, see [16,17]. We denote $\varphi = \frac{i}{2}\ln A$ and rearrange the product as $C = A\left(B + \frac{\partial \varphi}{\partial \omega}\frac{\partial B}{\partial t} - \frac{\partial \varphi}{\partial t}\frac{\partial B}{\partial \omega} - \frac{\partial \varphi}{\partial k}\frac{\partial B}{\partial r} + \frac{\partial \varphi}{\partial r}\frac{\partial B}{\partial k}\right)$. The gradient term in brackets can be viewed as a linear expansion of B with all arguments shifted as $C = A(\omega, k, r, t)B\left(\omega - \frac{\partial \varphi}{\partial t}, k + \frac{\partial \varphi}{\partial r}, r - \frac{\partial \varphi}{\partial k}, t + \frac{\partial \varphi}{\partial \omega}\right)$.

known from the theory of gases [3,13,14], however, the set of shifts is larger due to the medium effects on the binary collision that are dominated by the Pauli blocking of the internal states of the collision.

The physical meaning of the Δ's is best seen on gradually more complex limiting cases:

(o) Sending all Δ's to zero, (7) reduces to the BE (1).

(i) In the classical limit for hard spheres of the diameter d, the scattering phase shift $\phi \to \pi - |q|d$ gives $\Delta_4 = \Delta_2 = \frac{q}{|q|}d$ and all other Δ's are zero. The Enskog's nonlocal corrections are thus recovered.

(ii) For a collision of two isolated particles interacting via the single-channel separable potential, the scattering phase shift does not depend on r, t, q and $k - p$, while it depends on $k + p$ exclusively via the energy dependency, $\phi \to \phi(\Omega - \frac{1}{4m}(k + p)^2)$. Then $\Delta_{E,K,2} = 0$ and $\Delta_{3,4} = \frac{k+p}{2m}\Delta_t$. Since $\frac{k+p}{2m}$ is the center-of-mass velocity, the displacements $\Delta_4 = \Delta_3$ represent a distance over which particles fly together as a molecule.

(iii) For two isolated particles interacting via a general spherical potential, the scattering phase shift reflects the translational and the spherical symmetries. From translation of the center of mass during Δ_t follows the relation for the molecular flight $\frac{1}{2}(\Delta_4 + \Delta_3 - \Delta_2) = \frac{k+p}{2m}\Delta_t$. From the spherical symmetry follows that the sum of relative coordinates of the particles a and b at the end and at the beginning of collision has the direction of the transferred momentum $\frac{1}{2}(\Delta_4 - \Delta_3 + \Delta_2) = \frac{q}{|q|}d$ (Enskog-type shift), and that the difference has the perpendicular in-plane direction $\frac{1}{2}(\Delta_4 - \Delta_3 - \Delta_2) = \frac{k-p-q}{|k-p-q|}\alpha$ (rotation of the molecule). The nonlocality of the collision is thus given by three scalars Δ_t, d and α.

(iv) For effectively isolated two-particle collisions (no Pauli blocking but mean-field contributions $U_a(r, t)$ to the energy, i.e., $\phi \to \phi(\Omega - \frac{1}{4m}(k + p)^2 - U_a - U_b, \ldots)$) all Δ's become nonzero. The space displacements are the same as in (iii). The energy and the momentum corrections $2\Delta_{E,K} = -\Delta_t\frac{\partial}{\partial t, r}(U_a + U_b)$, represent the energy and the momentum which the effective molecule gains during its short life time Δ_t. These corrections are in fact of three-particle nature, however, only on the mean-field level, thus formally within the binary process. The three scalars, Δ_t, d and α, are still sufficient to parameterize all Δ's. We note that in the energy-conserving δ function all Δ's and mean-fields compensate $\delta\left(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 - 2\Delta_E\right) = \delta\left(\frac{k^2}{2m} + \frac{p^2}{2m} - \frac{(k-q)^2}{2m} - \frac{(p+q)^2}{2m}\right)$. This limit describes the dilute quantum gases and (7) reduces to the kinetic equation found in [13,14].

(v) With the medium effects on collisions, all Δ's become independent. A generally nonparabolic momentum-dependency of the quasiparticle energy does not allow to separate the center-of-mass motion, and an anisotropic,

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inhomogeneous and time-dependent distribution in the Pauli blocking ruins all symmetries of the collision. The energy conservation does not reduce to the simple conservation of the kinetic energy. The uncompensated residuum of the energy gain contributes to a conversion between the kinetic and the configuration energies of the system.

The kinetic equation (7) is numerically tractable by recent Monte Carlo codes. Kortemayer, Daffin and Bauer [6] have already studied the kinetic equation with an intuitive extension by Enskog-type displacement, claiming only a little increase in numerical demands compared to the BE. The larger set of Δ's in (7) does not require principal changes of the numerical method used in [6]. The functions ε, z, $|T^R_{sc}|^2$, and Δ's should form a consistent set, i.e., it is preferable to obtain them from Green's function studies, e.g., like [21]. Eventual fitting parameters should enter directly the effective nucleon-nucleon interaction.

IV. OBSERVABLES

Let us presume that a reasonable set of functions ε, z, $|T^R_{sc}|^2$, and Δ's is known. One can then proceed to evaluate observables. Here we present the density n_a of particles a, the density of energy $\mathcal{E}$, and the stress tensor $\mathcal{J}_{ij}$.

The observables in question are directly obtained from balance equations which also establish conservation laws. Integrating the kinetic equation (2) over momentum k with factors $\varepsilon_1, k, 1$, one finds that each observable has the standard quasiparticle part following from the drift

$$
\begin{aligned}
\mathcal{E}^{qp} &= \sum_a \int \frac{dk}{(2\pi)^3} \frac{k^2}{2m} f_1 \\
&+ \frac{1}{2} \sum_{a,b} \int \frac{dkdp}{(2\pi)^6} T_{ex}(\varepsilon_1 + \varepsilon_2, k, p, 0) f_1 f_2, \\
\mathcal{J}^{qp}_{ij} &= \sum_a \int \frac{dk}{(2\pi)^3} \left(k_j \frac{\partial \varepsilon_1}{\partial k_i} + \delta_{ij} \varepsilon_1 \right) f_1 - \delta_{ij} \mathcal{E}^{qp}, \\
n^{qp}_a &= \int \frac{dk}{(2\pi)^3} f_1,
\end{aligned} \tag{8}
$$

and the Δ-contribution following from the nonlocality of the scattering integral

$$
\begin{aligned}
\Delta\mathcal{E} &= \frac{1}{2} \sum_{a,b} \int \frac{dkdpdq}{(2\pi)^9} P(\varepsilon_1 + \varepsilon_2) \Delta_t, \\
\Delta\mathcal{J}_{ij} &= \frac{1}{2} \sum_{a,b} \int \frac{dkdpdq}{(2\pi)^9} P\left[(p+q)\Delta_4 + (k-q)\Delta_3 - p\Delta_2 \right], \\
\Delta n_a &= \sum_b \int \frac{dkdpdq}{(2\pi)^9} P\Delta_t,
\end{aligned} \tag{9}
$$

where $P = |T^R_{sc}|^2 2\pi\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4) f_1 f_2(1-f_3-f_4)$. The arguments denoted by numerical subscripts are identical to those used in (1), for all Δ's are explicit. The T-matrix T_{ex} used in the quasiparticle part of energy is the real part of the antisymmetrized Bethe-Goldstone T-matrix, $T^R_{ex} = (1 - \delta_{ab})T^R_{sc} + \delta_{ab}\sqrt{2}T^R_{sc}$. The actual observables are sum of the quasiparticle part and the Δ-correction.

In the low density limit, the Δ-contributions (9) become proportional to the square of density. Therefore they turn into the second order virial corrections. In the degenerated system, the density dependence cannot be expressed in the power-law expansion since the T-matrix, and consequently all Δ's, depend on the density. Nevertheless, we find it instructive to call the Δ-contribution the virial corrections because of their similar structure.

The total energy $\mathcal{E} = \mathcal{E}^{qp} + \Delta\mathcal{E}$ conserves within kinetic equation (2). This energy conservation law generalizes the result of Bornath, Kremp, Kraeft and Schlanges [19] restricted to non-degenerated systems. At degenerated systems, a new mechanism of energy conversion appears due to the medium effect on binary collisions. This mechanism can be seen writing the energy conservation $\frac{\partial}{\partial t}\mathcal{E} = \frac{\partial}{\partial t}\mathcal{E}^{qp} + \frac{\partial}{\partial t}\Delta\mathcal{E} = 0$ in a form

$$
\frac{\partial \mathcal{E}^{qp}}{\partial t} = \sum_a \int \frac{dk}{(2\pi)^3} \varepsilon \frac{\partial f_1}{\partial t} - \sum_{a,b} \int \frac{dkdpdq}{(2\pi)^9} P\Delta_E. \tag{10}
$$

The first term on the right hand side is the drift contribution to the energy balance. It is the only mechanism which appears in the absence of the virial corrections. The second term is the mean energy gain. It provides the conversion of the interaction energy controlled by two-particle correlations into energies of single-particle excitations. By this mechanism, the latent heat hidden in the interaction energy is converted into "thermal" excitations. The non-zero energy conversion, $\Delta_E \neq 0$, results from the time-dependency of the scattering phase shift on the quasiparticle distribution via the Pauli blocking of internal states. Accordingly, the energy conversion is a consequence of the in-medium effects.

The energy gain has its space counterpart in the momentum gain to the stress forces $\frac{\partial}{\partial r_j}\mathcal{J}_{ij}$. The energy density contributing to the stress tensor (8) has the gradient

$$
\frac{\partial \mathcal{E}^{qp}}{\partial r_i} = \sum_a \int \frac{dk}{(2\pi)^3} \varepsilon \frac{\partial f_1}{\partial r_i} - \sum_{a,b} \int \frac{dkdpdq}{(2\pi)^9} P\Delta_K. \tag{11}
$$

In the absence of the virial corrections (9), the first term of (11) combines together with the derivative of the first term in (8) into the standard quasiparticle contribution. In the presence of the virial corrections, the energy gain is necessary to obtain the correct momentum conservation law.

The density of energy $\mathcal{E} = \mathcal{E}^{qp} + \Delta\mathcal{E}$ given by (8) and (9) alternatively results from Kadanoff and Baym formula,

$$\mathcal{E} = \sum_a \int \frac{dk}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{1}{2} \left(\omega + \frac{k^2}{2m} \right) G^<(\omega, k, r, t), \quad (12)$$

with $G^<$ in the extended quasiparticle approximation (5). The particle density $n_a = n_a^{\rm qp} + \Delta n_a$ obtained from (5) via the definition, $n_a = \int \frac{d\omega}{2\pi} \frac{dk}{(2\pi)^3} G^<$, also confirms (8) and (9). The equivalence of these two alternative approaches confirms that the extended quasiparticle approximation is thermodynamically consistent with the nonlocal corrections to the scattering integral.

For equilibrium distributions, formulas (8) and (9) provide equations of state. Two known cases are worthy of comparison. First, the particle density $n_a = n_a^{\rm qp} + \Delta n_a$ is identical to the quantum Beth-Uhlenbeck equation of state [19,18], where $n_a^{\rm qp}$ is called the free density and Δn_a the correlated density. Second, the virial correction to the stress tensor has a form of the collision flux contribution known in the theory of moderately dense gases [3,4].

V. SUMMARY

In this Letter we have derived the kinetic equation (7) which is consistent with thermodynamic observables (8) and (9) up to the second order virial coefficient. The presented theory extends the theory of quantum gases [13,14] and non-ideal plasma [19] to degenerated system. The most important new mechanism is the energy conversion which follows from the medium effect on binary collisions.

The proposed corrections can be evaluated from known in-medium T-matrices and incorporated into existing Monte Carlo simulation codes, e.g., with the routine used in [6]. The nonlocal corrections to the scattering integral and corresponding second order virial corrections to thermodynamic observables enlighten the link between the kinetic equation approach and the hydrodynamical modeling. With expected progress in fits of the single-particle energy and other ingredients of the kinetic equation, the virial corrections can improve our ability to infer the equation of state from the heavy ion collision data.

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